Spring 2015 **Quiz 1** **Date: February 21**
K.Yaghi **Math 201- Section X** **Duration: 1 hour**

Problem 1 *(answer on page* 1 *of the booklet)*
Which of the following sequences converge, and which diverge? Find the limit of each convergent sequence. (7 *pts each*)

a) $a_n = (-1)^n \frac{n!}{7^n} e^{\cos\left(\frac{\pi}{n}\right)}$ b) $b_n = (\frac{n-3}{n-4})^n(1-\frac{9}{n^2})^n$ c) $c_n = (e^n - 1)^{1/n}$

Problem 2 (*answer on pages* 2 *&* 3 *of the booklet*)
Which of the following series converge, and which diverge? When possible find the sum of the series. (7 *pts each*)

a) $\sum_{n=1}^{\infty} \frac{3^n}{8^{n-1}} + \ln(1-\frac{1}{n^2})$ b) $\sum_{n=2}^{\infty} \frac{1}{(\ln n)^{\ln n}}$ c) $\sum_{n=3}^{\infty} \frac{(-1)^n \sin n}{\pi^{n+1}}$ d) $\sum_{n=2}^{\infty} \frac{\sqrt[n]{n}-1}{n^{0.2}}$ e) $\sum_{n=2}^{\infty} n^2(\frac{1}{n^2} - \ln(1+\frac{1}{n^2}))$

Problem 3 (*answer on page* 4 *of the booklet*)
Find the interval of convergence of the power series

$$\sum_{n=2}^{\infty} \frac{(-1)^n}{(\ln n)^2 4^n}(x+7)^n$$

For what values of x does the series converge absolutely? Conditionally? (20 *pts*)

Problem 4 (*answer on page* 5 *of the booklet*)

a) (5 *pts*) Prove that

$$\arctan x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} \qquad |x| < 1 \qquad (1)$$

b) (7 *pts*) Use taylor's theorem to prove that (1) is also true for $x = 1$, i.e.

$$\frac{\pi}{4} = \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1}$$

Problem 5 (*answer on page 6 and the last page of the booklet)*

a) (4 *pts*) Write a power series expansion for the function $f(x) = \frac{1}{\sqrt{1-x^2}}$ about the point $x = 0$. Also find the taylor polynomials $p2(x)$ and $p4(x)$ generated by $f(x)$ about the point $x = 0$.

b) (4 *pts*) Find the first 4 non-zero terms of the power series expansion of the function $g(x) = \arcsin x$ about the point $x = 0$.

c) (4 *pts*) Estimate $\arcsin(-0.1)$ by $p2(?)$. Then use the alternating series estimation theorem to estimate the resulting error. Does $p2(?)$ tend to be too small or too large?

Good Luck & Best Wishes

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